An Introduction to Description Logic II

Syntax and Semantics

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Syntax

General Remarks

- DL languages differ for the presence of **concept constructors** $\sqcap$, $\sqcup$, $\neg$, etc..
- Each DL language is identified by a **name** $\mathcal{AL}$, $\mathcal{ALE}$, $\mathcal{ALC}$, etc. that denotes the set of concept constructors present in the language.
- A language $\mathcal{L}'$ is said to be **more expressive** than a language $\mathcal{L}''$, if in $\mathcal{L}'$ can be expressed every concept expressible in $\mathcal{L}''$.
- If a language $\mathcal{L}'$ **syntactically expands** a language $\mathcal{L}''$, clearly it is more expressive than $\mathcal{L}''$.
- Nevertheless, due to **semantics** reasons, there are languages that are more expressive than others, even though they are not syntactical expansions.

Description Signature

A **description signature** is a tuple $\mathbf{D} = \langle N_I, N_A, N_R \rangle$, where

- N_I, a set of **individual names**;
  - Notation: $a, b, c, \ldots$
  - Examples: `John`, `Mary`, `Prague`, `MainSquare`,
- N_A a set of **concept names** (or atomic concepts);
  - Notation: $A, B, C, \ldots$
  - Examples: `Person`, `Female`, `Tall`, `Fat`, `Hight`,
- N_R a set of **role names** (or atomic roles)
  - Notation: $R_1, R_2, \ldots$
  - Examples: `hasChild`, `hasSister`, `hasNear`, `hasTemperature`,

Frame Languages

The name $\mathcal{FL}$ stands for **frame language** because it has more or less the same expressive power of frame-based systems; they were studied in the 80's.

C, D	$\longrightarrow$	A	atomic concept	$\mathcal{FL}_0$
		$C \sqcap D$	conjunction	$\mathcal{FL}_0$
		$\forall R.C$	value restriction	$\mathcal{FL}_0$
		$\exists R.\top$	restricted existential quantif.	$\mathcal{FL}^-$

Examples

An example of $\mathcal{FL}_0$ concept:

$$\texttt{Person} \sqcap \forall \texttt{hasChild}.\texttt{Male}$$

"person who has only sons (if he has children)"

An example of $\mathcal{FL}^-$ concept:

$$\texttt{Person} \sqcap \exists \texttt{hasChild}.\top$$

"person who has a child"

Attributive Languages

- The name $\mathcal{AL}$ stands for **Attributive Language**, began to be studied in the last 80's;
- $\mathcal{AL}$ marks the difference between frame-based systems and the new systems based on a **description of attributes and predicates**;
- Attributive Languages are the **expansions of $\mathcal{FL}^-$** by means of the following constructors:

C, D	$\longrightarrow$	$\neg A$	atomic complementation	$\mathcal{AL}$
		$C \sqcup D$	disjunction	$\mathcal{ALU}$
		$\exists R.C$	existential quantification	$\mathcal{ALE}$
		$\neg C$	complementation	$\mathcal{ALC}$

Examples

An example of $\mathcal{AL}$ concept:

$$\texttt{Person} \sqcap \forall \texttt{hasChild}.\neg \texttt{Male}$$

"person who has no sons"

An example of $\mathcal{ALU}$ concept:

$$\texttt{Person} \sqcap (\forall \texttt{hasChild}.\texttt{Male} \sqcup \forall \texttt{hasChild}.\texttt{Female})$$

"person who has either only sons or only daughters"

Examples

An example of $\mathcal{ALE}$ concept:

$$\texttt{Person} \sqcap \exists \texttt{hasChild.Male}$$

"person who has a son"

An example of $\mathcal{ALC}$ concept:

$$\texttt{Person} \sqcap \neg \forall \texttt{hasChild.Male}$$

"person who has not only sons"

Existential Languages

- The name $\mathcal{EL}$ stands for **Existential Language**;
- The interest for $\mathcal{EL}$ began in the **last ten years**;
- this interest is due to its **good computational behavior** and the fact that some **ontology can be defined** in $\mathcal{EL}$.

C, D	$\longrightarrow$	A	atomic concept	$\mathcal{EL}$
		$C \sqcap D$	disjunction	$\mathcal{EL}$
		$\exists R.C$	existential quantification	$\mathcal{EL}$
		$C \sqcup D$	disjunction	$\mathcal{ELU}$
		$\neg C$	complementation	$\mathcal{ELC}$

Examples

An example of $\mathcal{EL}$ concept:

$$\texttt{Person} \sqcap \exists \texttt{hasChild.Male}$$

"person who has a son"

An example of $\mathcal{ELU}$ concept:

$$\texttt{Person} \sqcap (\exists \texttt{hasChild.Male} \sqcup \exists \texttt{hasChild.Female})$$

"person who has either a son or a daughter"

An example of $\mathcal{ELC}$ concept:

$$\texttt{Person} \sqcap \neg \exists \texttt{hasChild.Male}$$

"person who has no son"

Cardinality restrictions

Also some kinds of **cardinality restrictions** on the range of roles are considered:

C, D	$\longrightarrow$	$\geq n\, R$	unqualified	
		$\leq n\, R$	number	$\mathcal{N}$
		$= n\, R$	restriction	
		$\geq n\, R.C$	qualified	
		$\leq n\, R.C$	number	$\mathcal{Q}$
		$= n\, R.C$	restriction	

Examples

An example of $\mathcal{ALN}$ concept:

$$\texttt{Person} \sqcap (\geq 3)\texttt{hasChild} \sqcap \forall\texttt{hasChild.Male}$$

"person who has at least three children and has only sons"

An example of $\mathcal{ALQ}$ concept:

$$\texttt{Person} \sqcap (\geq 3)\texttt{hasChild.Male}$$

"person who has at least three sons"

Other languages

Other languages **built up from concept constructors** are the ones obtained by adding constructors for:

- **nominals**,
- **concrete domains**.

C, D	$\longrightarrow$	$\{a_1, \ldots, a_m\}$	nominals	$\mathcal{O}$
		d	concrete domains	(D)

Examples

An example of $\mathcal{ALO}$ concept:

$$\texttt{Country} \sqcap \forall \texttt{hasBorderWith}\{\texttt{Germany},\texttt{Poland},\texttt{Slovakia},\texttt{Austria}\}$$

"Country which has borders only with Germany, Poland, Slovakia and Austria"

An example of $\mathcal{EL}(\mathbb{N})$ concept:

$$\texttt{Person} \sqcap \exists \texttt{hasAge.30} \sqcap \exists \texttt{hasChild.}(\texttt{Male} \sqcap \exists \texttt{hasAge.10})$$

"person who is 30 years old and has a 10 years old son"

Role-based languages

There are other languages that are defined by **modifying the behavior** of role:

- The language $\mathcal{S}$ is obtained from $\mathcal{ALC}$ by allowing **transitive roles**.
- If $\mathcal{L}$ is a language, then the language $\mathcal{LH}$ is obtained by allowing **inclusion axioms** between roles.
- If $\mathcal{L}$ is a language, then the language $\mathcal{LI}$ is obtained by allowing **inverse roles** R^{-}.
- If $\mathcal{L}$ is a language, then the language $\mathcal{LF}$ is obtained by allowing **functional roles**.
- If $\mathcal{L}$ is a language, then the language $\mathcal{LR}$ is obtained by allowing **intersection of roles** $R \circ P$, sometimes along with **conjunction**, **disjunction** and **negation** of roles.

Semantics

Interpratations

An **interpretation** is a pair

$$\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$$

where:

- $\Delta^{\mathcal{I}}$ is a nonempty set, called **domain**;
- $\cdot^{\mathcal{I}}$ is an **interpretation function** that assigns:
  - to each individual name $a \in N_I$ an element
  $$a^{\mathcal{I}} \in \Delta^{\mathcal{I}},$$
  - to each atomic concept A a subset of the domain set
  $$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}},$$
  - to each role name R a binary relation on the domain set
  $$R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}.$$

Set theoretical fragment

The fragment of DL languages with just **propositional connectives** can be managed with set-theoretical basic operations on the domain:

$$
\begin{aligned}
\bot^{\mathcal{I}} &= \emptyset \\
\top^{\mathcal{I}} &= \Delta^{\mathcal{I}} \\
(\neg C)^{\mathcal{I}} &= \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}} \\
(C \sqcap D)^{\mathcal{I}} &= C^{\mathcal{I}} \cap D^{\mathcal{I}} \\
(C \sqcup D)^{\mathcal{I}} &= C^{\mathcal{I}} \cup D^{\mathcal{I}}
\end{aligned}
$$

Modal fragment

The fragment of DL languages with **quantifiers** can be managed with modal-like semantics:

$$
\begin{aligned}
(\exists R.\top)^{\mathcal{I}} &= \{v \in \Delta^{\mathcal{I}} \colon \text{exists } w \in \Delta^{\mathcal{I}} \text{ such that } R^{\mathcal{I}}(v, w)\} \\
(\forall R.C)^{\mathcal{I}} &= \{v \in \Delta^{\mathcal{I}} \colon \text{for every } w \in \Delta^{\mathcal{I}}, \\
&\quad \text{if } R^{\mathcal{I}}(v, w) \text{ then } C^{\mathcal{I}}(w)\} \\
(\exists R.C)^{\mathcal{I}} &= \{v \in \Delta^{\mathcal{I}} \colon \text{exists } w \in \Delta^{\mathcal{I}} \text{ such that} \\
&\quad R^{\mathcal{I}}(v, w) \text{ and } C^{\mathcal{I}}(w)\}
\end{aligned}
$$

Bounded quantifiers

The concept constructors denoting **cardinality restrictions** can be managed with bounded quantifiers:

$$
\begin{aligned}
(\geq n\, R)^{\mathcal{I}} &= \{v \in \Delta^{\mathcal{I}} \colon |\{b \in \Delta^{\mathcal{I}} \colon R^{\mathcal{I}}(v, w)\}| \geq n\} \\
(\leq n\, R)^{\mathcal{I}} &= \{v \in \Delta^{\mathcal{I}} \colon |\{b \in \Delta^{\mathcal{I}} \colon R^{\mathcal{I}}(v, w)\}| \leq n\} \\
(= n\, R)^{\mathcal{I}} &= \{v \in \Delta^{\mathcal{I}} \colon |\{b \in \Delta^{\mathcal{I}} \colon R^{\mathcal{I}}(v, w)\}| = n\} \\
(\geq n\, R.C)^{\mathcal{I}} &= \{v \in \Delta^{\mathcal{I}} \colon |\{b \in \Delta^{\mathcal{I}} \colon R^{\mathcal{I}}(v, w) \wedge C^{\mathcal{I}}(w)\}| \geq n\} \\
(\leq n\, R.C)^{\mathcal{I}} &= \{v \in \Delta^{\mathcal{I}} \colon |\{b \in \Delta^{\mathcal{I}} \colon R^{\mathcal{I}}(v, w) \wedge C^{\mathcal{I}}(w)\}| \leq n\} \\
(= n\, R.C)^{\mathcal{I}} &= \{v \in \Delta^{\mathcal{I}} \colon |\{b \in \Delta^{\mathcal{I}} \colon R^{\mathcal{I}}(v, w) \wedge C^{\mathcal{I}}(w)\}| = n\}
\end{aligned}
$$

Other concept constructors

- The semantics of **nominal concepts** is the following:

$$\{a_1, \ldots, a_m\}^{\mathcal{I}} = \{a_1^{\mathcal{I}}, \ldots, a_m^{\mathcal{I}}\} \subseteq \Delta^{\mathcal{I}}$$

- **Concrete domains** allow to consider data coming from a domain D different from the abstract domain $\Delta^{\mathcal{I}}$. Usually the sets $\mathbb{N}$ or $\mathbb{R}$ are considered, with their **natural semantics**. Indeed, the use of concrete domains is due to the fact that they usually have their own semantics and there is **no need of an ontology** to state the relations among the domain elements.

Semantics of complex roles

The definition of a concept-like semantics for role constructor is possible just for the following constructors:

$$
\begin{aligned}
(R^{-})^{\mathcal{I}} &= \{(b,a) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \colon (a,b) \in R^{\mathcal{I}}\} \\
(\neg R)^{\mathcal{I}} &= \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \setminus R^{\mathcal{I}} \\
(R \sqcap S)^{\mathcal{I}} &= R^{\mathcal{I}} \cap S^{\mathcal{I}} \\
(R \sqcup S)^{\mathcal{I}} &= R^{\mathcal{I}} \cup S^{\mathcal{I}} \\
(R \circ S)^{\mathcal{I}} &= R^{\mathcal{I}} \circ S^{\mathcal{I}}
\end{aligned}
$$

Restrictions on roles

The semantics of **transitive** and **functional** roles is given by means of general restrictions:

- a role R is **transitive** if $R \circ R$ is equal to R,
- A role R is **functional** if

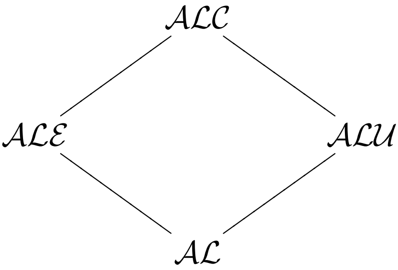
$$\forall v, w, z \in \Delta^{\mathcal{I}}, \quad R^{\mathcal{I}}(v, w) \wedge R^{\mathcal{I}}(v, z) \quad \Longrightarrow \quad w = z$$

Role hierarchies just admit inclusion axioms between roles.

Language Hierarchies

Inclusions between languages: the $\mathcal{ALC}$ hierarchy

A straightforward consequence of the semantics of constructors is that every $\mathcal{ALE}$ and every $\mathcal{ALU}$ concepts are $\mathcal{ALC}$ concepts, but there are $\mathcal{ALE}$ concepts that are not $\mathcal{ALU}$ concepts and vice-versa. So, the hierarchy of languages between $\mathcal{AL}$ and $\mathcal{ALC}$ appears as follows



Other inclusions

Let $\mathcal{L}$ a DL language, then:

- language $\mathcal{LQ}$ includes language $\mathcal{LN}$:

$$(\geq n)R.\top$$

- language $\mathcal{LQ}$ includes language $\mathcal{LF}$:

$$(\leq 1)R.C$$

- **role composition** can be defined in $\mathcal{EL}$

$$\exists(R \circ P).C \equiv \exists R.\exists P.C$$