Complexity sources in Fuzzy Description Logics

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The three main continuous t-norms

	Minimum (Gödel)	Product	Łukasiewicz
$x * y$	$\min(x, y)$	$x \cdot y$	$\max(0, x + y - 1)$
$x \Rightarrow y$	$\begin{cases} 1, & \text{if } x \leq y \\ y, & \text{otherwise} \end{cases}$	$\begin{cases} 1, & \text{if } x \leq y \\ y/x, & \text{otherwise} \end{cases}$	$\min(1, 1 - x + y)$
$x \Rightarrow 0$	$\begin{cases} 1, & \text{if } x = 0 \\ 0, & \text{otherwise} \end{cases}$	$\begin{cases} 1, & \text{if } x = 0 \\ 0, & \text{otherwise} \end{cases}$	$1 - x$

Semantics

$$
\begin{aligned}
(\sim C)^{\mathcal{I}}(v) &:= 1 - C^{\mathcal{I}}(v) \\
(C \sqcap D)^{\mathcal{I}}(v) &:= C^{\mathcal{I}}(v) * D^{\mathcal{I}}(v) \\
(C \sqcup D)^{\mathcal{I}}(v) &:= 1 - ((1 - C^{\mathcal{I}}(v)) * (1 - D^{\mathcal{I}}(v))) \\
(C \to D)^{\mathcal{I}}(v) &:= C^{\mathcal{I}}(v) \Rightarrow D^{\mathcal{I}}(v) \\
(\forall R.C)^{\mathcal{I}}(v) &:= \inf_{w \in \Delta^{\mathcal{I}}} \{R^{\mathcal{I}}(v, w) \Rightarrow C^{\mathcal{I}}(w)\} \\
(\exists R.C)^{\mathcal{I}}(v) &:= \sup_{w \in \Delta^{\mathcal{I}}} \{R^{\mathcal{I}}(v, w) * C^{\mathcal{I}}(w)\}
\end{aligned}
$$

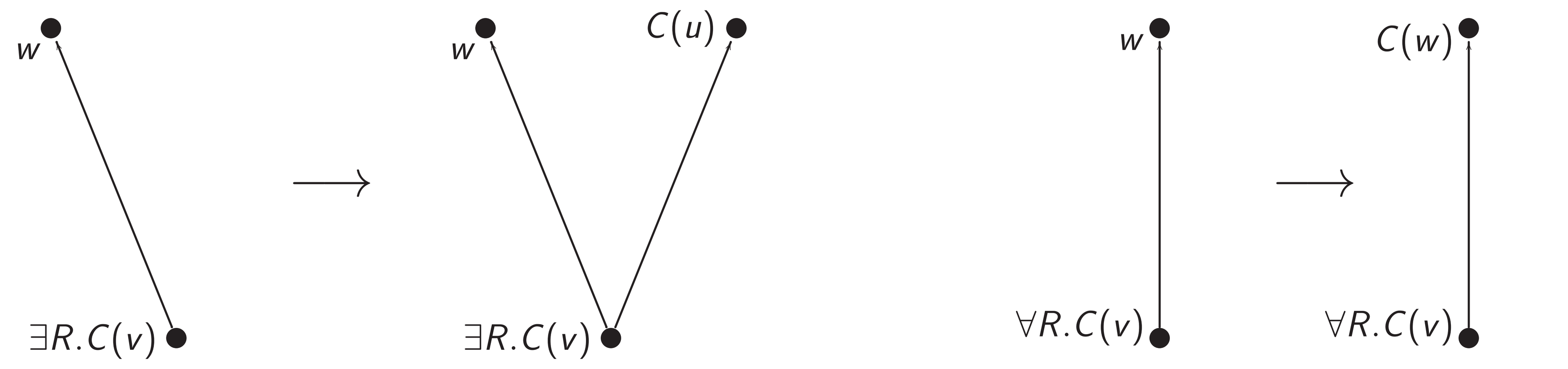
Behavior of tableau-like algorithms for lower bound reasoning tasks

The sets of truth values for a given classical axiom or reasoning task can be taken from a range included between a positive value $r > 0$ and 1. That is, the graded axioms have the following form:

$$\langle C \sqsubseteq D \geq r \rangle,$$

$$\langle C(a) \geq r \rangle.$$

The classical tableau algorithm adds a new element just when it finds out an existentially quantified subconcept $\exists R.C$, but not for value restrictions $\forall R.C$:

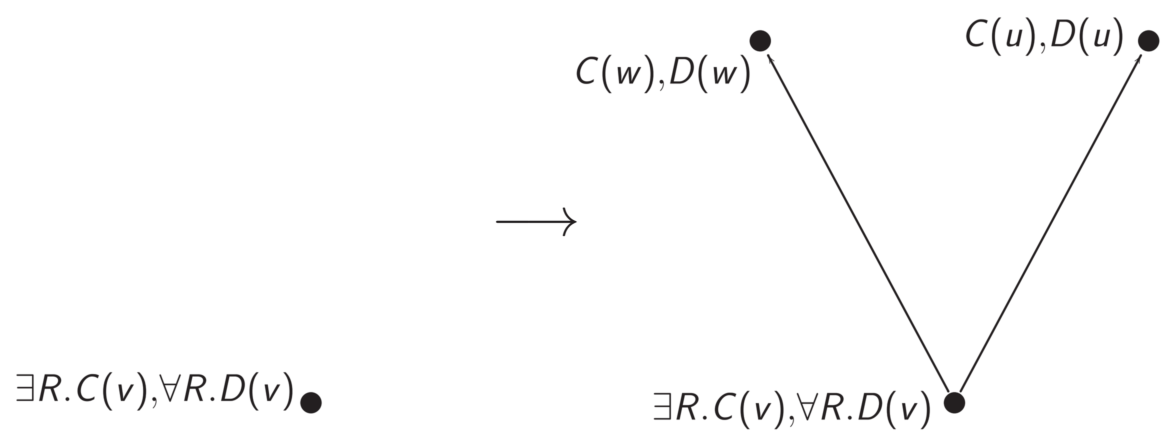


Behavior of tableau-like algorithms for exact-value reasoning tasks

Assertion axioms and concept satisfiability can be asked to take single values only, different than 1, then having the following form:

$$\langle C(a) = r \rangle.$$

The main difference is that tableau-like algorithms for exact-value reasoning tasks add a new element not only when an existentially quantified subconcept $\exists R.C$ is found, but also when a value restriction $\forall R.C$ has to be computed.



Classical structural subsumption algorithm $SUBS?[C, D]$ from [Brachman and Levesque, 1984]

1: Flatten both C and D by removing all nested $\sqcap$ operators.

2: Collect all arguments to an $\forall R.$ for a given role R.

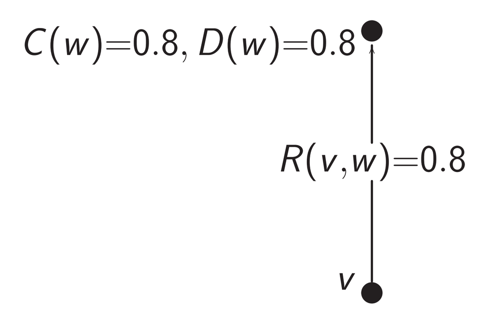
3: Assuming that $C := C_1 \sqcap \ldots \sqcap C_n$ and $D := D_1 \sqcap \ldots \sqcap D_m$, then return **true** iff for each C_i:

(a) if C_i is an atom or a $\exists R.\top$, then one of D_j is C_i.

(b) if C_i is $\forall R.E$ then one of the D_j is $\forall R.F$, where $SUBS?[E, F]$.

Non-idempotent conjunction

Under non-idempotent conjunction, concepts $\forall R.(C \sqcap D)$ and $\forall R.C \sqcap \forall R.D$ are not equivalent:



Hence, step 2 of algorithm $SUBS?[C, D]$ can not be applied.

Structural algorithm Ł$_n$-$SUBS(1, D, C)$ for 1-subsumption in Ł$_n$-$\mathcal{FL}^-$

if there is an occurrence of an atomic or existential conjunct A of D that is not in C where concept A appears in C strictly less $n - 1$ times **then**
    return 0
else
    $\mathbf{E}_{C,D} := \emptyset$
    for all value restriction $\forall R.F$ which is a conjunct of D **do**
        for all value restriction $\forall R.E$ which is a conjunct of C **do**
            $\mathbf{E}_{C,D}(\forall R.F, \forall R.E) :=$ Ł$_\mathbf{n}$-$SUBS(1, F, E)$
        end for
    end for
    if there is a maximal bipartite matching for $\mathbf{E}_{C,D}$ **then**
        return 1
    else
        return 0
    end if
end if

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