Theoretical foundations of fuzzy description logics and their application as languages for ontology representation

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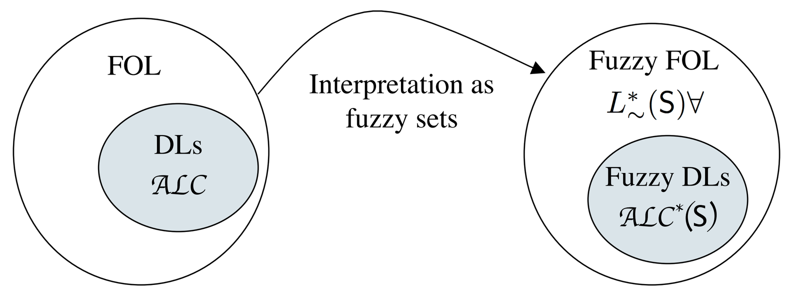
The aim of this doctoral thesis is a systematic development of expressive fuzzy Description Logics taking as a starting point current researches on first order fuzzy logics and modal fuzzy logics.

This point of view was firstly proposed by [1] and then developed in [2] where some research lines to this end are proposed.



References

[1] Hájek, P. 2005. Making fuzzy description logic more general. *Fuzzy Sets and Systems* 154(1):1-15.

[2] García–Cerdaña, A.; Armengol, E. and Esteva, F. 2010. Fuzzy Description Logics and t-norm based Fuzzy Logics. *International Journal of Approximate Reasoning*. doi: 10.1016/j.ijar.2010.01.001.

1. Logical Background

From a logical point of view, we are interested in studying the first order fuzzy logic associated to the consequence relation of the **standard semantics** (the one with truth values in [0,1]) of the three basic t-norms, Ł, Π and G and **finite-valued logics** like $Ł_n$ and G_n, because these are more suited of being used in real applications. For the same reason we are also interested in their expansions by an appropriate set of **truth constants** (S) or by an **involutive negation** $\sim$.
For each of the logics above mentioned we want to know whether they enjoy the so-called **witnessed model property** and **quasi-witnessed model property**, because, through these properties, it is possible to prove **decidability** for their $\mathcal{ALC}$-like fragments.

$$\text{w/qw-}L\text{-}P \iff L\text{-}P \iff [0,1]_L\text{-}P$$

Where $P \in \{Sat, 1\text{-}Sat, pos\text{-}Sat\}$ and L is one among the standard first order logics above mentioned.

	$Taut^w$	$1\text{-}Sat^w$	$pos\text{-}Sat^w$	$Taut^{qw}$	$1\text{-}Sat^{qw}$	$pos\text{-}Sat^{qw}$
$[0,1]_Ł$	Y [3,2]	Y [3,2]	Y [3,2]	Y [3,2]	Y [3,2]	Y [3,2]
$[0,1]_\Pi$	N [2]	N [2]	N [2]	Y [1,4]	?	Y [1,4]
$[0,1]_G$	N [2]	N [2]	N [2]	N [2]	N [2]	N [2]
$Ł_n$	Y	Y	Y	Y	Y	Y
G_n	Y	Y	Y	Y	Y	Y
$[0,1]_Ł(S)$	?	?	?	?	?	?
$[0,1]_\Pi(S)$	N [2]	N [2]	N [2]	?	?	?
$[0,1]_{\Pi,\sim}$	?	?	?	?	?	?

References

[1] Cerami, M.; Esteva F. and Bou F. 2010. Decidability of a Description Logic over infinite-valued Product Logic. KR 2010 Conference.

[2] Cintula, P. and Hájek, P. 2006. On theories and models in fuzzy predicate logic. *Journal of Symbolic Logic* 71(3):863-880.

[3] Hájek, P. 1998. *Metamathematics of Fuzzy Logic.* Dordrecht: Kluwer Academic Publishers.

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2. A new framework for FDLs

FDLs defined in the 90s are mainly based on Zadeh semantics (for example [2]) and sometimes in Lukasiewicz first order logic. The introduction of other t-norms deserves **new symbols**. In [1] we propose for an $\mathcal{ALC}$-like language the following set of concepts constructors:

- $\boxplus$ strong union $\mathcal{U}$
- $\boxtimes$ strong intersection $\mathcal{AL}$
- $\sqsupset$ residuated implication prefixed $\mathcal{I}$
- $\sim$ involutive complement $\mathcal{C}$

We will keep the symbols $\sqcap$, $\sqcup$, and $\neg$ to denote the constructors associated to weak intersection, weak union and residuated complement, as well as $\top$ and $\bot$ for the domain and empty concept respectively.
In the same paper [1], for each fuzzy logic, we propose a new hierarchy of $\mathcal{ALC}$-like Description Languages that fits better inside the framework of a t-norm-based treatment of many-valued Description Logic.

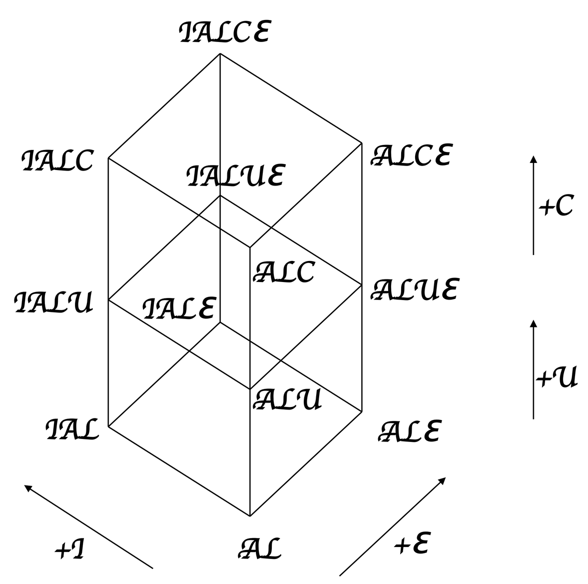

Figure 2: *Hierarchy of languages*

References

[1] Cerami, M.; García–Cerdaña, A. and Esteva, F. 2010. From Classical Description Logic to n-graded Fuzzy Description Logic. Accepted at FUZZ-IEEE 2010 Conference.

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3. Decidability

First order standard tautologies are **very complex**: for instance, they are not recursively enumerable in the case of infinite-valued Lukasiewicz Logic and not arithmetical in the case of infinite-valued Product Logic. Nevertheless, the $\mathcal{ALC}$-like fragments have been shown to be decidable.

	$Taut$	$1\text{-}Sat$	$pos\text{-}Sat$
Ł-$\mathcal{ALC}$	[3,4]	[3,4]	[3,4]
Π-$\mathcal{IALE}$	[2]	[2]*	[2]
G-$\mathcal{IALE}$	?	?	?
$Ł_n$-$\mathcal{ALC}$	[3]**	[3]**	[3]**
G_n-$\mathcal{IALE}$	[3]**	[3]**	[3]**
Ł(S)-$\mathcal{ALC}$	?	?	?
$\Pi(S)$-$\mathcal{IALE}$	?	?	?
Π-$\mathcal{IALCE}$	[1]***	[1]***	[1]***

* Only for its axiomatic extension complete with respect to quasi-witnessed models.
** In [3] there is not an explicit proof of decidability of finite-valued $\mathcal{ALC}$-like FDLs, but the proof provided there can be used for finite-valued logics.
*** Only for its axiomatic extension complete with respect to witnessed models.

References

[1] Bobillo, F. and Straccia, U. 2009. Fuzzy description logics with general t-norms and datatypes *Fuzzy Sets and Systems* 160:3382-3402.

[2] Cerami, M.; Esteva F. and Bou F. 2010. Decidability of a Description Logic over infinite-valued Product Logic. KR 2010 Conference.

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4. Future work

In the future we will focus our work on the following subjects:

1. Logical Background
   - Providing results about witnessed and quasi-witnessed model properties for expansions of first order fuzzy logics with truth value constants.
   - Studying the relationship between Fuzzy Description Logic and Fuzzy Modal Logic (see [2]).
2. Decidability and Algorithms
   - Providing missing algorithms for expansions of logics with truth value constants.
   - Providing algorithms for languages more expressive than $\mathcal{ALC}$.
   - Studying the computational complexity of the provided algorithms.
   - A comparative study of the different proposed algorithms, like in [1] or [3].
3. Applications
   A final goal will be providing working tools able to represent fuzzy ontologies and reason with vague concepts.

References

[1] Bobillo, F. and Straccia, U. 2009. Fuzzy description logics with general t-norms and datatypes *Fuzzy Sets and Systems* 160:3382-3402.

[2] Bou, F.; Esteva, F.; Godo, L. and Rodriguez, R. On the Minimum Many-Valued Modal Logic over a Finite Residuated Lattice. *Journal of Logic and Computation.* doi: 10.1093/logcom/exp062.

[3] Stoilos, G.; Stamou, G.; Tzouvaras, J. Z. P. V. and Horrocks, I. 2007. Reasoning with very expressive Fuzzy Description Logics. *Journal of Artificial Intelligence Research* 30:273-320.

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