call INSERT, \#S (P :X 379/200 :Y 27/250)
structure view:

data view:

call CHOOSE-SUBTREE with 0, node root, level 0
the node root is not full, add the record.

call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
call INSERT, \#S (P :X 313/200 :Y 401/125)
structure view:

| 0 |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |

data view:

| 1 |  |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
| 0 |  |

call CHOOSE-SUBTREE with 1 , node root, level 0
the node root is not full, add the record.

call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
call INSERT, \#S (P :X 2333/1000 :Y 67/250)
structure view:

$$
\begin{array}{|l|l|l|l|l|}
\hline 0 & 1 & & & \\
\hline
\end{array}
$$

data view:

call CHOOSE-SUBTREE with 2, node root, level 0
the node root is not full, add the record.

call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
call INSERT, \#S (P :X 639/1000 :Y 2531/1000)
structure view:

$$
\begin{array}{|l|l|l|l|l|}
\hline 0 & 1 & 2 & & \\
\hline
\end{array}
$$

data view:

call CHOOSE-SUBTREE with 3, node root, level 0
the node root is not full, add the record.

$$
\begin{array}{|l|l|l|l|l}
\hline 0 & 1 & 2 & & \\
\hline
\end{array} \quad \rightarrow \quad \longrightarrow \quad \begin{array}{|l|l|l|l|l|}
\hline 0 & 1 & 2 & 3 & \\
\hline
\end{array}
$$

call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
call INSERT, \#S (P : X 13/10 :Y 3087/1000)
structure view:

$$
\begin{array}{|l|l|l|l|l|}
\hline 0 & 1 & 2 & 3 & \\
\hline
\end{array}
$$

data view:

call CHOOSE-SUBTREE with 4, node root, level 0
the node root is not full, add the record.

$$
\begin{array}{|l|l|l|l|l|}
\hline 0 & 1 & 2 & 3 & \\
\hline
\end{array} \quad \longrightarrow \quad \begin{array}{|l|l|l|l|l|}
\hline 0 & 1 & 2 & 3 & 4 \\
\hline
\end{array}
$$

call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
call INSERT, \#S (P : X 192/125 : Y 153/100)
structure view:

$$
\begin{array}{|l|l|l|l|l|}
\hline 0 & 1 & 2 & 3 & 4 \\
\hline
\end{array}
$$

data view:

call CHOOSE-SUBTREE with 5 , node root, level 0 call OVERFLOW-TREATMENT call REINSERT


The two most distant nodes (20) are removed and reinserted in reversed order.
The rest is kept.
Adjust MBR of root.
Reinserting 0 .
call CHOOSE-SUBTREE with 0 , node $A$, level 0
the node A is not full, add the record.

$$
\begin{array}{|l|l|l|l|l}
\hline 1 & 3 & 4 & 5 & \\
\hline
\end{array} \quad \longrightarrow \quad \quad \quad \begin{array}{|l|l|l|l|l|}
\hline 0 & 1 & 3 & 4 & 5 \\
\hline
\end{array}
$$

call ADJUST-TREE with $R$, node A
we are at the root
return from ADJUST-TREE
return from REINSERT
Reinserting 2.
call CHOOSE-SUBTREE with 2 , node $A$, level 0
call OVERFLOW-TREATMENT
Second overflow on the same level during one insert - make a split.
call SPLIT-NODE with nodes A and 2
call CHOOSE-SPLIT-AXIS with ...
Axis: $x$
Entries sorted by low: 345102
Entries sorted by high: 345102

$o=5.9139996$
partition by low: $345-102$

partition by low: $3451-02$

$o=4.002$
partition by high: $34-5102$

partition by high: $345-102$

partition by high: $3451-02$


Axis: $y$
Entries sorted by low: 025341
Entries sorted by high: 025341

partition by low: $025-341$

partition by high: $025-341$

partition by low: $0253-41$


Minimal margin ( $o=4.002$ ) was reached for axis $x$. return from CHOOSE-SPLIT-AXIS
call CHOOSE-SPLIT-INDEX with entries, axis $\mathrm{x}, R$
Entries sorted by low at axis $x$ : 345102
Entries sorted by high at axis $x$ : 345102

Partition by low: $34-5102$


Partition by low: $345-102$

overlap $=0.300447$ $S=5.1218286$

Partition by low: $3451-02$


Partition by high: $34-5102$


Partition by high: $345-102$


$$
\begin{gathered}
\text { overlap }=0.300447 \\
S=5.1218286
\end{gathered}
$$

Partition by high: $3451-02$

... and the winner is:

return from CHOOSE-SPLIT-INDEX.
call ADJUST-TREE with $R$, node A and the new node
we are at the root
return from ADJUST-TREE
create a new root. return from REINSERT
call ADJUST-TREE with $R$, node A update MBR of node A.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root we are at the root
return from ADJUST-TREE
structure view:

data view:

call CHOOSE-SUBTREE with 6 , node root, level 1 Next level are leaf nodes


B

old overlap $=0$
new overlap $=0.0039450294$
overlap extension $=0.0039450294$ area extension $=1.089675$

Node $A$ is chosen
the node A is not full, add the record.

$$
\begin{array}{|l|l|l|l|l}
\hline 1 & 3 & 4 & 5 & \\
\hline
\end{array} \quad \longrightarrow \quad \begin{array}{|l|l|l|l|l|}
\hline 1 & 3 & 4 & 5 & 6 \\
\hline
\end{array}
$$

call ADJUST-TREE with $R$, node A
update MBR of node A.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
call INSERT, \#S (P :X 51/125 : Y 349/200)
structure view:

data view:

call CHOOSE-SUBTREE with 7, node root, level 1 Next level are leaf nodes


B

old overlap $=0$ new overlap $=0.5440652$
overlap extension $=0.5440652$ area extension $=3.6739445$

Node $A$ is chosen call OVERFLOW-TREATMENT
call REINSERT


The two most distant nodes (61) are removed and reinserted in reversed order.
The rest is kept.
Adjust MBR of A.
Reinserting 1.
call CHOOSE-SUBTREE with 1, node root, level 1
Next level are leaf nodes


Node $A$ is chosen
the node A is not full, add the record.

$$
\begin{array}{|l|l|l|l|l|}
\hline 3 & 4 & 5 & 7 & \\
\hline
\end{array} \quad \longrightarrow \quad \begin{array}{|l|l|l|l|l|}
\hline 1 & 3 & 4 & 5 & 7 \\
\hline
\end{array}
$$

call ADJUST-TREE with $R$, node A
update MBR of node A.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
return from REINSERT
Reinserting 6.
call CHOOSE-SUBTREE with 6 , node root, level 1
Next level are leaf nodes


Node $A$ is chosen
call OVERFLOW-TREATMENT
Second overflow on the same level during one insert - make a split.
call SPLIT-NODE with nodes A and 6
call CHOOSE-SPLIT-AXIS with ...
Axis: $x$
Entries sorted by low: 734516
Entries sorted by high: 734516

$o=3.9449997$
partition by low: $734-516$

$o=4.9259997$
partition by low: $7345-16$
 $o=5.285$
partition by high: $73-4516$

partition by high: $734-516$

partition by high: $7345-16$


Axis: $y$
Entries sorted by low: 567341
Entries sorted by high: 567341
partition by high: $56-7341$


$$
o=3.697
$$

partition by low: $567-341$

$o=3.96$


Minimal margin ( $o=3.529$ ) was reached for axis $y$. return from CHOOSE-SPLIT-AXIS
call CHOOSE-SPLIT-INDEX with entries, axis y, $R$
Entries sorted by low at axis $y$ : 567341
Entries sorted by high at axis y: 567341
partition by high: $567-341$


Partition by low: 56-7341

overlap $=0.010992027$ $S=2.365573$

Partition by low: $567-341$

overlap $=0$
$S=1.6274321$
Partition by low: $5673-41$

overlap $=0$
$S=2.0012069$

Partition by high: $56-7341$


$$
\text { overlap }=0.010992027
$$

$$
S=2.365573
$$

Partition by high: $567-341$


Partition by high: $5673-41$

overlap $=0$
$S=2.0012069$
... and the winner is:

return from CHOOSE-SPLIT-INDEX.
call ADJUST-TREE with $R$, node A and the new node update MBR of node $A$.
add the new node to the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
return from REINSERT
call ADJUST-TREE with $R$, node C
update MBR of node C.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
structure view:

data view:

call CHOOSE-SUBTREE with 8, node root, level 1
Next level are leaf nodes
A

old overlap $=0$
new overlap $=0$
overlap extension $=0$
area extension $=0.6111227$
B

old overlap $=0$
new overlap $=0.03735002$
overlap extension $=0.03735002$
area extension $=1.3686947$

old overlap $=0$ new overlap $=0$ overlap extension $=0$ area extension $=0.93442584$

Node $A$ is chosen
the node A is not full, add the record.

$$
\begin{array}{|l|l|l|l|l|}
\hline 1 & 3 & 4 & & \\
\hline
\end{array} \quad \longrightarrow \quad \longrightarrow \quad \begin{array}{|l|l|l|l|l|}
\hline 1 & 3 & 4 & 8 & \\
\hline
\end{array}
$$

call ADJUST-TREE with $R$, node A
update MBR of node A.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
structure view:

data view:

call CHOOSE-SUBTREE with 9, node root, level 1
Next level are leaf nodes

old overlap $=0$
new overlap $=0.6399301$
overlap extension $=0.6399301$
area extension $=2.8692155$

old overlap $=0$ new overlap $=0$ overlap extension $=0$ area extension $=1.5119749$

old overlap $=0$
new overlap $=0$ overlap extension $=0$ area extension $=1.2590581$

Node $C$ is chosen
the node C is not full, add the record.

$$
\begin{array}{|l|l|l|l|l|}
\hline 5 & 6 & 7 & & \\
\hline
\end{array} \quad \longrightarrow \quad \begin{array}{|l|l|l|l|l|}
\hline 5 & 6 & 7 & 9 & \\
\hline
\end{array}
$$

call ADJUST-TREE with $R$, node C
update MBR of node C.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
structure view:

data view:

call CHOOSE-SUBTREE with 10, node root, level 1
Next level are leaf nodes


old overlap $=0$
new overlap $=0$
overlap extension $=0$
area extension $=0.21501005$

old overlap $=0$
new overlap $=0.22967997$
overlap extension $=0.22967997$
area extension $=3.1029139$

Node $B$ is chosen
the node B is not full, add the record.

call ADJUST-TREE with $R$, node B
update MBR of node B.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
call INSERT, \#S (P : X 117/50 :Y 1/1000)
structure view:

data view:

call CHOOSE-SUBTREE with 11, node root, level 1
Next level are leaf nodes

A

old overlap $=0$
new overlap $=1.8381122$
overlap extension $=1.8381122$
area extension $=4.8780813$

B

old overlap $=0$
new overlap $=0$
overlap extension $=0$
area extension $=0.122715026$

old overlap= 0
new overlap $=0.23606995$
overlap extension $=0.23606995$
area extension $=2.2689476$

Node $B$ is chosen
the node B is not full, add the record.

call ADJUST-TREE with $R$, node B
update MBR of node B.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
structure view:

data view:

call CHOOSE-SUBTREE with 12, node root, level 1
Next level are leaf nodes

old overlap $=0$ new overlap $=0$
overlap extension $=0$ area extension $=0.0$

old overlap $=0$
new overlap $=0.2917751$
overlap extension $=0.2917751$
area extension $=3.1539002$

old overlap $=0$ new overlap $=0.86596996$
overlap extension $=0.86596996$ area extension $=1.6658509$

Node $A$ is chosen
the node A is not full, add the record.

call ADJUST-TREE with $R$, node A
update MBR of node A.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
call INSERT, \#S (P : X 3621/1000 :Y 1243/500)
structure view:

data view:

call CHOOSE-SUBTREE with 13, node root, level 1
Next level are leaf nodes

old overlap $=0$
new overlap $=0$
overlap extension $=0$
area extension $=1.9811244$

old overlap $=0$
new overlap $=0.13370511$
overlap extension $=0.13370511$
area extension $=4.6039047$

old overlap $=0$ new overlap $=0.57266284$
overlap extension $=0.57266284$
area extension $=4.824287$

Node $A$ is chosen call OVERFLOW-TREATMENT
call REINSERT


The two most distant nodes (13 3) are removed and reinserted in reversed order.
The rest is kept.
Adjust MBR of A.
Reinserting 3.
call CHOOSE-SUBTREE with 3, node root, level 1
Next level are leaf nodes


Node $A$ is chosen
the node A is not full, add the record.

$$
\begin{array}{|l|l|l|l|l|}
\hline 1 & 4 & 8 & 12 & \\
\hline
\end{array} \quad \longrightarrow \quad \begin{array}{|l|l|l|l|l|}
\hline 1 & 3 & 4 & 8 & 12 \\
\hline
\end{array}
$$

call ADJUST-TREE with $R$, node A
update MBR of node A.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
return from REINSERT
Reinserting 13.
call CHOOSE-SUBTREE with 13, node root, level 1
Next level are leaf nodes


old overlap $=0$
new overlap $=0.57266284$
overlap extension $=0.57266284$ area extension $=4.824287$

Node $A$ is chosen
call OVERFLOW-TREATMENT
Second overflow on the same level during one insert - make a split.
call SPLIT-NODE with nodes A and 13
call CHOOSE-SPLIT-AXIS with ...
Axis: $x$
Entries sorted by low: 34112813
Entries sorted by high: 34112813
partition by low: $34-112813$

$o=4.9979997$
partition by low: $341-12813$

partition by low: $34112-813$

$o=4.6309996$
partition by high: $34-112813$

partition by high: $341-12813$

partition by high: $34112-813$


Axis: $y$
Entries sorted by low: 81331241
Entries sorted by high: 81331241
partition by low: $813-31241$


$$
o=4.6309996
$$


partition by low: $8133-1241$

$o=5.0609994$


Minimal margin ( $o=4.5909996$ ) was reached for axis $y$. return from CHOOSE-SPLIT-AXIS
call CHOOSE-SPLIT-INDEX with entries, axis y, $R$ Entries sorted by low at axis y: 81331241
Entries sorted by high at axis $y$ : 81331241
partition by high: $8133-1241$

partition by high: $813312-41$


Partition by low: $813-31241$

overlap $=0.026194999$
$S=2.0093127$

Partition by low: $8133-1241$

overlap $=0.018224724$ $S=1.9372928$

Partition by low: $813312-41$

overlap $=0$
$S=2.1316507$

Partition by high: $813-31241$

overlap $=0.026194999$
$S=2.0093127$

Partition by high: $8133-1241$

overlap $=0.018224724$ $S=1.9372928$

Partition by high: $813312-41$

overlap $=0$
$S=2.1316507$
... and the winner is:

return from CHOOSE-SPLIT-INDEX.
call ADJUST-TREE with $R$, node A and the new node update MBR of node $A$.
add the new node to the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
return from REINSERT
call ADJUST-TREE with $R$, node D
update MBR of node D.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
structure view:

data view:

call CHOOSE-SUBTREE with 14, node root, level 1
Next level are leaf nodes

A

old overlap $=0$
new overlap $=0.913317$
overlap extension $=0.913317$ area extension $=1.8671289$

old overlap $=0$
new overlap $=0.06044507$
overlap extension $=0.06044507$ area extension $=1.6451101$

D

old overlap $=0$
new overlap $=0.42345312$
overlap extension $=0.42345312$
area extension $=2.1033025$

Node $C$ is chosen
the node C is not full, add the record.

$$
\begin{array}{|l|l|l|l|l}
\hline 5 & 6 & 7 & 9 & \\
\hline
\end{array} \quad \longrightarrow \quad \longrightarrow \quad \begin{array}{|l|l|l|l|l|}
\hline 5 & 6 & 7 & 9 & 14 \\
\hline
\end{array}
$$

call ADJUST-TREE with $R$, node C
update MBR of node C.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
structure view:

data view:

call CHOOSE-SUBTREE with 15 , node root, level 1
Next level are leaf nodes

old overlap $=0$ new overlap $=0$
overlap extension $=0$ area extension $=0.54415477$

old overlap $=0$
new overlap $=4.173024$
overlap extension $=4.173024$
area extension $=9.936571$
D

old overlap= 0
new overlap $=0.14926499$
overlap extension $=0.14926499$
area extension $=2.5549813$

Node $A$ is chosen
the node A is not full, add the record.

call ADJUST-TREE with $R$, node A update MBR of node A.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
structure view:

data view:

call CHOOSE-SUBTREE with 16, node root, level 1
Next level are leaf nodes

A

old overlap $=0$
new overlap $=3.1146884$
overlap extension $=3.1146884$ area extension $=5.83674$

B

old overlap $=0$ new overlap $=0$
overlap extension $=0$
area extension $=0.023084999$
D

old overlap $=0$
new overlap $=2.407295$
overlap extension $=2.407295$
area extension $=6.3512716$

Node $B$ is chosen
the node B is not full, add the record.

$$
\begin{array}{|l|l|l|l|l}
\hline 0 & 2 & 10 & 11 & \\
\hline
\end{array} \quad \rightarrow \quad \begin{array}{|l|l|l|l|l|}
\hline 0 & 2 & 10 & 11 & 16 \\
\hline
\end{array}
$$

call ADJUST-TREE with $R$, node B update MBR of node B.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
structure view:

data view:

call CHOOSE-SUBTREE with 17, node root, level 1
Next level are leaf nodes

A

old overlap $=0$
new overlap $=2.4098738$
overlap extension $=2.4098738$ area extension $=5.8537555$

old overlap $=0$
new overlap $=0.64133406$
overlap extension $=0.64133406$ area extension $=3.0459028$

D

old overlap $=0$
new overlap $=0.39559013$
overlap extension $=0.39559013$
area extension $=1.7787385$

Node $C$ is chosen
call OVERFLOW-TREATMENT
call REINSERT


The two most distant nodes (917) are removed and reinserted in reversed order.
The rest is kept.
Adjust MBR of C.
Reinserting 17.
call CHOOSE-SUBTREE with 17, node root, level 1
Next level are leaf nodes


B
 new overlap $=0.21039604$
overlap extension $=0.21039604$ area extension $=3.0459028$

D

old overlap $=0$
new overlap $=0.39559013$
overlap extension $=0.39559013$
area extension $=1.7787385$

Node $C$ is chosen
the node C is not full, add the record.

$$
\begin{array}{|l|l|l|l|l}
\hline 5 & 6 & 7 & 14 & \\
\hline
\end{array}
$$

call ADJUST-TREE with $R$, node C
update MBR of node C.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
return from REINSERT
Reinserting 9.
call CHOOSE-SUBTREE with 9 , node root, level 1
Next level are leaf nodes


Node $B$ is chosen
call OVERFLOW-TREATMENT
Second overflow on the same level during one insert - make a split.
call SPLIT-NODE with nodes B and 9
call CHOOSE-SPLIT-AXIS with ...
Axis: $x$
Entries sorted by low: 901621110
Entries sorted by high: 901621110
partition by low: $90-1621110$

$o=4.168$
partition by high: $90-1621110$

$o=4.168$
partition by low: $9016-21110$

partition by low: $90162-1110$

$o=4.023$
Axis: y
Entries sorted by low: 111002169
Entries sorted by high: 111002169

partition by low: $11100-2169$

$o=4.314$
partition by high: $9016-21110$


$$
o=3.805
$$

partition by high: $90162-1110$

partition by high: $1110-02169$

partition by high: $11100-2169$

partition by low: $111002-169$

$o=4.064$
Minimal margin ( $o=3.805$ ) was reached for axis $x$. return from CHOOSE-SPLIT-AXIS
call CHOOSE-SPLIT-INDEX with entries, axis $\mathrm{x}, R$
Entries sorted by low at axis $x$ : 901621110
Entries sorted by high at axis $x$ : 901621110

Partition by low: $90-1621110$

overlap $=0.057987$
$S=1.952333$
Partition by low: $9016-21110$

overlap $=0$
$S=1.7858491$
Partition by low: $90162-1110$

overlap $=0.03744197$
$S=1.9734249$
partition by high: $111002-169$


Partition by high: $90-1621110$

overlap $=0.057987$
$S=1.952333$
Partition by high: $9016-21110$


Partition by high: $90162-1110$

overlap $=0.03744197$
$S=1.9734249$
$\ldots$ and the winner is:

return from CHOOSE-SPLIT-INDEX.
call ADJUST-TREE with $R$, node B and the new node update MBR of node B.
add the new node to the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
return from REINSERT
call ADJUST-TREE with $R$, node C update MBR of node C.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root we are at the root
return from ADJUST-TREE
structure view:

data view:

call CHOOSE-SUBTREE with 18, node root, level 1
Next level are leaf nodes

old overlap $=0$ new overlap $=0$ overlap extension $=0$ area extension $=0.25484687$

old overlap $=0$
new overlap $=1.6811292$
overlap extension $=1.6811292$ area extension $=4.429276$

old overlap $=0$ new overlap $=2.5305205$ overlap extension $=2.5305205$ area extension $=5.5770774$

D

old overlap $=0$
new overlap $=0.47291988$
overlap extension $=0.47291988$ area extension $=2.1955796$

E

old overlap $=0$
new overlap $=2.230959$ overlap extension $=2.230959$ area extension $=4.667058$

Node $A$ is chosen
the node A is not full, add the record.

call ADJUST-TREE with $R$, node A update MBR of node A.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
structure view:

data view:

call CHOOSE-SUBTREE with 19, node root, level 1
Next level are leaf nodes

old overlap $=0$
new overlap $=0.016055795$
overlap extension $=0.016055795$
area extension $=0.3595587$

old overlap $=0$
new overlap $=3.161229$
overlap extension $=3.161229$
area extension $=6.219045$

old overlap= 0 new overlap $=1.9715636$ overlap extension $=1.9715636$ area extension $=3.8813215$

D

old overlap $=0$
new overlap $=0.00936596$
overlap extension $=0.00936596$
area extension $=0.5982158$

old overlap $=0$ new overlap $=1.6670461$
overlap extension $=1.6670461$ area extension $=3.7905662$

Node $D$ is chosen
the node D is not full, add the record.

$$
\begin{array}{|l|l|l|l|l}
\hline 3 & 8 & 12 & 13 & \\
\hline
\end{array} \quad \rightarrow \quad \begin{array}{|l|l|l|l|l|}
\hline 3 & 8 & 12 & 13 & 19 \\
\hline
\end{array}
$$

call ADJUST-TREE with $R$, node D update MBR of node D.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
structure view:

data view:

call CHOOSE-SUBTREE with 20, node root, level 1
Next level are leaf nodes
A

old overlap $=0.00936596$
new overlap $=4.0519266$
overlap extension $=4.0425606$ area extension $=7.921458$

old overlap $=0$
new overlap $=0$
overlap extension $=0$ area extension $=0.0$

old overlap $=0$ new overlap $=1.744989$ overlap extension $=1.744989$ area extension $=5.0298414$

D

old overlap $=0.00936596$ new overlap $=2.872095$ overlap extension $=2.862729$ area extension $=7.1340437$

E

old overlap $=0$
new overlap $=0.117851906$
overlap extension $=0.117851906$
area extension $=0.5288758$

Node $B$ is chosen
the node B is not full, add the record.

call ADJUST-TREE with $R$, node B update MBR of node B.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
structure view:

data view:

call CHOOSE-SUBTREE with 21, node root, level 1
Next level are leaf nodes
A

old overlap $=0.00936596$
new overlap $=0.010703863$
overlap extension $=0.0013379026$
area extension $=0.054392815$
B

old overlap $=0$
new overlap $=4.919038$
overlap extension $=4.919038$
area extension $=9.817356$

old overlap $=0$
new overlap $=2.9229534$
overlap extension $=2.9229534$
area extension $=5.2270293$

D

old overlap $=0.00936596$
new overlap $=0.37073678$
overlap extension $=0.36137083$ area extension $=1.2408276$

old overlap $=0$
new overlap $=2.30928$
overlap extension $=2.30928$
area extension $=5.349328$

Node $A$ is chosen
the node A is not full, add the record.

| 1 | 4 | 15 | 18 |  |
| :--- | :--- | :--- | :--- | :--- |$\quad \longrightarrow \quad$| 1 | 4 | 15 | 18 | 21 |
| :--- | :--- | :--- | :--- | :--- |

call ADJUST-TREE with $R$, node A update MBR of node A.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
structure view:

data view:

call CHOOSE-SUBTREE with 22 , node root, level 1
Next level are leaf nodes


Node $D$ is chosen
call OVERFLOW-TREATMENT
call REINSERT


The two most distant nodes (313) are removed and reinserted in reversed order.
The rest is kept.
Adjust MBR of D.
Reinserting 13.
call CHOOSE-SUBTREE with 13, node root, level 1
Next level are leaf nodes


Node $D$ is chosen
the node D is not full, add the record.

call ADJUST-TREE with $R$, node D
update MBR of node D.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
return from REINSERT
Reinserting 3.
call CHOOSE-SUBTREE with 3, node root, level 1
Next level are leaf nodes


Node $D$ is chosen
call OVERFLOW-TREATMENT
Second overflow on the same level during one insert - make a split.
call SPLIT-NODE with nodes D and 3
call CHOOSE-SPLIT-AXIS with ...
Axis: $x$
Entries sorted by low: 3191282213
Entries sorted by high: 3191282213
partition by low: $319-1282213$

$o=3.911$
partition by high: $319-1282213$

$o=3.911$
partition by low: $31912-82213$


$$
o=4.507
$$

partition by low: $319128-2213$


$$
o=3.0959997
$$

Axis: y
Entries sorted by low: 2281331219
Entries sorted by high: 2281331219

partition by low: $22813-31219$

$o=4.507$
partition by high: $31912-82213$

$o=4.507$
partition by high: $319128-2213$


$$
o=3.0959997
$$

partition by high: $228-1331219$

partition by high: $22813-31219$

partition by low: $228133-1219$

$o=5.2549996$
Minimal margin ( $o=3.0959997$ ) was reached for axis $x$. return from CHOOSE-SPLIT-AXIS
call CHOOSE-SPLIT-INDEX with entries, axis $\mathrm{x}, R$
Entries sorted by low at axis $x$ : 3191282213
Entries sorted by high at axis x: 3191282213

Partition by low: $319-1282213$

overlap $=0$

$$
S=1.9252051
$$

Partition by low: $31912-82213$

overlap $=0.026194999$ $S=1.945443$

Partition by low: $319128-2213$

overlap $=0$
$S=1.3131337$
partition by high: $228133-1219$


Partition by high: $319-1282213$


$$
\text { overlap }=0
$$

$$
S=1.9252051
$$

Partition by high: $31912-82213$


$$
\begin{gathered}
\hline \text { overlap }=0.026194999 \\
S=1.945443
\end{gathered}
$$

Partition by high: $319128-2213$

overlap $=0$
$S=1.3131337$
... and the winner is:

return from CHOOSE-SPLIT-INDEX.
call ADJUST-TREE with $R$, node D and the new node update MBR of node D.
add the new node to the parent node root
Parent node root is full, promote split (create a new parent) call SPLIT-NODE with nodes root and NIL call CHOOSE-SPLIT-AXIS with ...
Axis: $x$
Entries sorted by low: A E C D B F
Entries sorted by high: A D E B C F
partition by low: A E - C D B F

$o=12.125$
partition by low: A E C - D B F

$o=13.537999$
partition by high: A D - E B C F

$o=9.471999$
partition by high: A D E-B C F

$o=11.716999$
partition by low: A E C D-B F

$o=11.435999$
Axis: y
Entries sorted by low: B E C F D A
Entries sorted by high: B E C F D A

partition by low: B E C - F D A

$o=10.646999$
partition by low: B E C F - D A

$o=9.471999$
Minimal margin ( $o=9.471999$ ) was reached for axis $x$. return from CHOOSE-SPLIT-AXIS
call CHOOSE-SPLIT-INDEX with entries, axis $\mathrm{x}, R$ Entries sorted by low at axis $x$ : A E C D B F
partition by high: A D E B - C F

$o=11.263$
partition by high: B E - C F D A

$o=9.512$
partition by high: B E C - F D A

$o=10.646999$
partition by high: B E C F - D A

$o=9.471999$

Partition by low: A E - C D B F

overlap $=5.1777244$
$S=17.989336$
Partition by low: A E C - D B F

overlap $=9.396942$ $S=23.009125$

Partition by low: A E C D - B F

overlap $=3.7458337$
$S=17.16248$

Partition by high: A D - E B C F

overlap $=0.67059184$ $S=11.886461$

Partition by high: A D E - B C F

overlap $=4.4702525$ $S=16.596833$

Partition by high: A D E B - C F

overlap $=3.123512$
$S=15.086332$
... and the winner is:

return from CHOOSE-SPLIT-INDEX.
continue by adjusting the parent node NIL, the new parent
call ADJUST-TREE with $R$, node B and the new node
we are at the root
return from ADJUST-TREE
create a new root. return from REINSERT
call ADJUST-TREE with $R$, node D update MBR of node $D$.
continue by adjusting the parent node B
call ADJUST-TREE with $R$, node B
update MBR of node B.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
structure view:

data view:

call CHOOSE-SUBTREE with 23 , node root, level 2
Next level are not leaf nodes

overlap extension $=4.4353743$
area extension $=4.4353743$

overlap extension $=2.8774045$
area extension $=2.8774045$

Node $H$ is chosen
call CHOOSE-SUBTREE with 23 , node $H$, level 1
Next level are leaf nodes


Node $A$ is chosen call OVERFLOW-TREATMENT
call REINSERT


The two most distant nodes (23 21 ) are removed and reinserted in reversed order.
The rest is kept.
Adjust MBR of A.
Reinserting 21.
call CHOOSE-SUBTREE with 21, node root, level 2
Next level are not leaf nodes

overlap extension $=3.319724$
area extension $=3.319724$


Node $H$ is chosen
call CHOOSE-SUBTREE with 21, node $H$, level 1
Next level are leaf nodes


Node $A$ is chosen
the node A is not full, add the record.

call ADJUST-TREE with $R$, node A
update MBR of node A.
continue by adjusting the parent node H
call ADJUST-TREE with $R$, node H
update MBR of node H .
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
return from REINSERT
Reinserting 23.
call CHOOSE-SUBTREE with 23 , node root, level 2
Next level are not leaf nodes


Node $H$ is chosen call CHOOSE-SUBTREE with 23 , node $H$, level 1
Next level are leaf nodes


Node $A$ is chosen
call OVERFLOW-TREATMENT
Second overflow on the same level during one insert - make a split. call SPLIT-NODE with nodes A and 23
call CHOOSE-SPLIT-AXIS with ...
Axis: $x$
Entries sorted by low: 2115411823
Entries sorted by high: 2115411823
partition by low: $2115-411823$

$o=3.6079996$
partition by high: $2115-411823$

$o=3.6079996$
partition by low: $21154-11823$

$o=4.4079995$
partition by low: $211541-1823$

$o=4.2729993$
Axis: $y$
Entries sorted by low: 2141151823
Entries sorted by high: 2141151823
partition by low: $214-1151823$

$o=5.6389995$
partition by low: $2141-151823$

$o=5.9259997$
partition by high: $21154-11823$

$o=4.4079995$
partition by high: $211541-1823$


$$
o=4.2729993
$$

partition by high: $214-1151823$

partition by high: $2141-151823$

partition by low: $214115-1823$

$o=4.2729993$
Minimal margin ( $o=3.6079996$ ) was reached for axis $x$. return from CHOOSE-SPLIT-AXIS
call CHOOSE-SPLIT-INDEX with entries, axis $\mathrm{x}, R$
Entries sorted by low at axis $x$ : 2115411823
Entries sorted by high at axis $x$ : 2115411823

Partition by low: $2115-411823$


$$
\text { overlap }=0
$$

$$
S=1.9066224
$$

Partition by low: $21154-11823$

overlap $=0$
$S=1.960293$
Partition by low: $211541-1823$

overlap $=0$
$S=1.6235383$
partition by high: $214115-1823$

$o=4.2729993$

Partition by high: $2115-411823$


$$
\text { overlap }=0
$$

$$
S=1.9066224
$$

Partition by high: $21154-11823$

overlap $=0$
$S=1.960293$
Partition by high: $211541-1823$

overlap $=0$
$S=1.6235383$
$\ldots$ and the winner is:

return from CHOOSE-SPLIT-INDEX.
call ADJUST-TREE with $R$, node A and the new node update MBR of node A.
add the new node to the parent node H
call ADJUST-TREE with $R$, node H update MBR of node H .
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root we are at the root return from ADJUST-TREE return from REINSERT
call ADJUST-TREE with $R$, node I
update MBR of node I.
continue by adjusting the parent node H
call ADJUST-TREE with $R$, node H update MBR of node H .
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE
structure view:

data view:

call CHOOSE-SUBTREE with 24 , node root, level 2
Next level are not leaf nodes


H

overlap extension $=4.702086$
area extension $=4.702086$

Node $G$ is chosen
call CHOOSE-SUBTREE with 24 , node $G$, level 1
Next level are leaf nodes

old overlap $=0$
new overlap $=0.23716511$
overlap extension $=0.23716511$ area extension $=0.956573$

old overlap $=0$
new overlap $=0.1283161$
overlap extension $=0.1283161$
area extension $=2.300418$
F

old overlap $=0$
new overlap $=0.82455916$
overlap extension $=0.82455916$
area extension $=3.458187$

Node $E$ is chosen
the node E is not full, add the record.

call ADJUST-TREE with $R$, node E
update MBR of node E .
continue by adjusting the parent node G
call ADJUST-TREE with $R$, node G
update MBR of node G.
continue by adjusting the parent node root
call ADJUST-TREE with $R$, node root
we are at the root
return from ADJUST-TREE


